



THEORETICAL PREDICTIONS FOR VLF RADIO PROPAGATION

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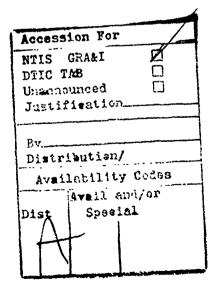
In this Technical Report, an attempt has been made to make theoretical computations of the vertical components of the individual and multimode field strengths as a function of distance based on mode theory. The variations of various ionospheric parameters, e.g., attenuation rates, the heights of the ionospheric reflection point, the height gain factors for an appropriate combination of the transmitting and receiving antenna elevations, along with the presence of the earth's geomagnetic field, especially for the East-West propagation, have been duly considered. The results obtained have been presented in tabular and graphical forms and are consistent with the values obtained by earlier workers. These field strength values will be compared against the experimental values when the above-mentioned ambitious experiment is carried out in the Pacific in the late Summer of 1983.

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ABSTRACT

The very low frequency (VLF, 3 to 30 KHz) part of the radio frequency spectrum is characterized by low attenuation rate, high phase and frequency stability, and high signal to noise ratio. Consequently, VLF radio propagation is used for many practical applications, e.g., frequency standardization, clock synchronization, and reliable long-distance radio communications. Because of the distinct advantages of VLF radio propagation, the U.S. Navy will be conducting a balloon-to-balloon-borne cross link communication experiment to study the characteristics of the VLF high altitude radio propagation.

In this technical report, an attempt has been made to make theoretical computations of the vertical components of the individual and multimode field strengths as a function of distance based on mode theory. The variations of various ionospheric parameters, e.g., attenuation rates, the height of the ionospheric reflection point, the height gain factors for an appropriate combination of the transmitting and receiving antenna elevations, along with the presence of the earth's geomagnetic field, especially for the East-West propagation, have been duly considered. The results obtained have been presented in tabular and graphical form and are consistent with the values obtained by earlier workers. These field strength values will be compared against the experimental values when the above-mentioned ambitious experiment is carried out in the Pacific in the late Summer of 1983.

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INTRODUCTION

Analogous to microwave propagation along a waveguide, very low frequency (VLF, 3 to 30 KHz) radio propagation over long distance is treated as if the propagation takes place in a terrestrial spherical waveguide with boundaries formed by the conducting earth's surface and the lower edge of the reflecting ionosphere. For the frequency range under consideration, (15 to 30 KHz, i.e., the upper band of the VLF spectrum) the analysis of the wave propagation is simplified because of the limited number of significant modes that need to be considered. The level of excitation of the individual modes depends upon the corresponding excitation factors and the variation of field strengths across the waveguide cross-section as given by the so-called height gain functions. For lower order modes (especially say, the first order mode) the energy of the wave is mostly concentrated at the base of the ionosphere. For such a mode, the excitation factor is small, which implies inefficiency for launching such a wave from a ground-based point as the excitation factor for such a wave will not depend much on ground conductivity. However, as it turns out, the height gain factors for such waves increase as the height of the transmitting antenna increases. The waves become earth detached and are called earth-detached modes (nomenclature due to James R. Wait) or whispering gallery modes (nomenclature due to K. G. Budden). Such waves, naturally characterized with very low attenuation rates, criss-cross the ionosphere without any intermediate earth-based bouncing point and propagate over long distances without losing much of their energy.

The U.S. Navy is planning a balloon-to-balloon communication link experiment employing this mode of propagation. The plans for this experiment have been given elsewhere (3). In this technical report, only the theoretically computed values of the vertical field components of the field strength radiated out by a vertically polarized antenna at different distances for different configurations of the transmitting and receiving antenna heights will be presented.

THEORY

The theory of very low frequency radio propagation has been thoroughly discussed in many well known texts and research reports (2,4,5,6) and (2,4,5,6) and (2,4,5,6) and (2,4,5,6) and (2,4,5,6) are several approximation theories can be utilized in order to limit the mathematical sophistication within bounds. In this consideration, the earth-ionosphere terrestrial waveguide is considered as a thin spherical shell. Spherical coordinate system (1,0,0) is used and the boundary conditions at the two interfaces are defined by their impedance conditions, i.e.,

$$z_g = \frac{E}{H_o}$$
; $z_i = -\frac{E}{H_o}$

where

 $z_g = impedance of the conducting ground$

 $Z_{+} = impedance of the conducting ionosphere$

 E_A = tangential electric field component

 $H\phi = zonal$ magnetic field component

Furthermore, for special interest, discussion will be restricted to two discrete frequencies, 19.4 and 27 kHz, as these are the two frequencies which will be operational during the balloon-to-balloon-borne cross link communication experiment. Two levels of transmitter power will be used, one kilowatt and 440 watts. (For details, please refer to (3)). The ionosphere will be characterized with an exponential profile having a conductivity gradient $\ell=2$ (or $\ell=0.5$), where is the length over which ionospheric conductivity, which is a measure of the ratio ω_0/ν or N/ ν , changes by a ratio of 2.71, where ω_0 is the plasma frequency and ν is the collisonal frequency and ν is the electron density in the ionospheric layer.

According to the formulations of WKB (or second order) approximations, the integral form of the modal equation or the resonance condition for mode propagation of spherical waves can be written as (2,4,6)

$$R_{g}(C_{n}) R_{i}(C_{n}') \exp(-2ik \int_{0}^{h} (C_{n} + 2z/a) dz + i\pi/2)$$

where $R_{i}(C_{n})$ and $R_{i}(C_{n})$ are the Fresnel reflection coefficients of the earth's surface and the ionospheric surface for the complex angles of incidences;

 $c_n = (1 - s_n^2)^{1/2}$; $c'_n = (1 - s_n^2)^{1/2}$

 $= \exp(-2in\pi)$

(1)

The other constants are defined as follows:

 $k = propagation constant = 2\pi / \lambda$ a = radius of the earth surface

$$z_0 = -ac_n^2/2$$

As stated by Wait (4), under this approximation, valid for the frequency range of our interest (20 to 30 kHz), the modal characteristics are independent of the electrical properties of the earth's surface and the modes are characterized as earth-detached mode in analogy to their counterparts in accoustics originally investigated by Lord Rayleigh at the turn of the present century.

The total phase given by the integral form of the modal equation consists of a "to and fro" path between the ionospheric reflection point at z=h and a second reflecting point at z = z a height embraced by a surface, called caustic surface, below which the waves of lower order modes becomes evanescent. The phase shift at the caustic accounts for $\pi/2$. The excitation factors for these modes for the ground-based launching points are very small.

Assuming earth's surface as a perfect electric conductor and the ionospheric surface as a perfect magnetic conductor (i.e., $R_g = 1$ and $R_f = -1$), equation (1) reduces to

$$-2ik \int_{0}^{h} (c_{n}^{2} + 2z/a) dz + i\pi /2 = -2ni \pi + i\pi$$
 (2)

and the solution is:

$$C_n = (\frac{3\pi}{ka} (n - 1/4))^{2/3} - 2h/a$$

where C_n specifies the resonance condition for the modes.

The expression for the amplitude of the vertical component of the electric field strength at a certain distance can be written as (2,6)

$$E_{z} = \frac{300}{h} \left(\frac{P_{R} \times \lambda}{a \sin D/a} \right)^{1/2} \sum_{n} \Lambda_{n} G_{n}(z) G_{n}(z)$$
(3)

$$\exp (-\alpha_n D)$$
 volt/meter

and the phase of the nth mode component relative to the antenna current is given by (6)

$$\phi_{E_{z,n}} = \pi/4 - \omega D/v_{p,n} + \phi_{\Lambda_n}$$
 (4)

where E₂ = Modesum field strength amplitude in volt/meter

h = ionospheric height in meters

 λ = c/f = wavelength in meters

P_R = radiated power in kilowatts

D = distance in meters

 Λ_{-n} = nth mode excitation factor

Gn(z) = height gain factor for the transmitting antenna at
 height z meters

Gn(2) * height gain factor for the receiving antenna at
height z meters

on = nth mode attenuation constant in dB/megameter

 $E_{z,n}$ = phase of the nth mode in radians

 $\phi \cdot An$ = phase of the nth mode excitation factor in radians

 ω = angular frequency of the wave in radians

 $v_{p,n}$ = phase velocity of the nth mode in meters/second.

As noted earlier, for the frequency range under consideration (15-30 kHz), very few modes are important for consideration. Consequently, for the present prupose only the first three modes (mode 1 through 3) will be considered.

SINGLE MODE FIELD STRENGTH

We can express the field strength amplitude in dBv/m from equation (3) for the nth mode as (where n is either 1 or 2 or 3):

$$E_{zn} = 20 \text{ Log (300)} - 20 \text{ Log (h)} + 10 \text{ Log } P_R + 10 \text{ Log (3X10}^8)$$

$$-10 \text{ Log (f)} - 10 \text{ Log (a sin D/a)} - \frac{\alpha_n D}{10^6}$$

+ HGF dBv/m (5)

= 20 Log (
$$10^6$$
) + 20 Log (300) - 20 Log (h)

+10 Log (P_R) + 10 Log ($3X10^8$) - 10 Log (f)

-10 Log (a sin D/a) - $\frac{\alpha_n D}{10^6}$

+ HGF
$$dB\mu v/m$$
 (6)

For daytime condition (h = 70 km) and for frequency f = 19.4 kHZ and $P_R = 1$ kilowatt, equation (6) reduces to:

$$E_{z,n} = 54.3 - 10 \text{ Log (a sin D/a)} - \alpha D$$
+ HGF $dB\mu\nu/m.kw$ (7)

and for nighttime condition (h = 90 km)

$$E_{z,n} = 53.1 - 10 \text{ Log (a sin D/a)} - \alpha D$$

+ HGF dBuv/m.kw (8)

Similarly, for the second frequency, f = 27 kHZ and for the day and night-time conditions, the corresponding expressions are:

$$E_{z,n} = 52.3 - 10 \text{ Log (a sin D/a)} - \alpha \tilde{I}$$

+ HGF dB $\mu\nu/m.kw$ (9)
(f = 27 kHZ, h = 70 km)

and

$$E_{z,n} = 50.9 - 10 \text{ Log(a sin D/a)} - \alpha D + HGF dB \mu v/m.kw$$
 (10)

(f=27 kHz, h=90 km)

In equations (7) through (10), a and D are expressed in megameters and α is in dB/megameter (Mm). HGF is the modified height gain function for the nth order mode and is given by (2)

$$HGF = 20 \text{ Log } (\Lambda_n G_n(z) G_n(\hat{z})) dB$$
 (11)

Following reference (2) we can write

$$\Lambda_{n} = \frac{y_{0}}{2 t_{n}^{1/2} (y_{0} - t_{n})} \exp \left(-\frac{4}{3} t_{n}^{3/2}\right)$$
 (12)

$$G_n(z) = \sqrt{\pi} \quad t_n^{1/4} A_i(t_n - z) \exp \left(+ \frac{2}{3} t_n^{3/2} \right)$$
 (13)

and

$$G_{n}(\hat{z}) = \sqrt{\pi} t_{n}^{1/4} A_{1}(t_{n} - \hat{z}) \exp(+\frac{2}{3}t_{n}^{3/2})$$
 (14)

where

$$y_{0} = (ka/2) \quad (2h/a)$$

$$z = (ka/2) \quad (h_{T}/a)$$

$$z = (ka/2) \quad (h_{R}/a)$$

$$z = (ka/2) \quad (-c_{n}^{2})$$

$$c_{n}^{2} = (\frac{3\pi}{\kappa a} (n - 1/4))^{2/3} - 2h/a$$

 h_T = height of the transmitting antenna above the ground surface h_R = height of the receiving antenna above the ground surface $A_i(t_n - z)$ and $A_i(t_n - z)$ are the Airy functions of arguments $(t_n - z)$ and $(t_n - z)$ as defined in reference (8).

The arguments (t_{n-2}) or (t_{n-2}) , since they are symmetrical, of the Airy functions can be simplified as follows:

$$t_{n-z} = \frac{(ka)^{2/3}}{2} \left[\frac{2h}{a} - \left[\frac{3II}{ka} (n-1/4) \right]^{2/3} - \frac{2h_T}{a} \right]$$

$$= \frac{(2)}{a} \frac{(ka)^{2/3}}{2} (h-h_T) - \frac{(3II)^{2/3}}{2} (n-1/4)^{2/3}$$

or

$$(t_{n-z}) = 5.15 \times 10^{-3} (h-h_T) f^{2/3} - 2.81 (n-1)^{2/3}$$
 (15)

where h and hT are in km and f in kHz.

Similarly for t_{n-z} , we can write:

$$(t_{n-z}) = 5.15 \times 10^{-3} (h-h_R) f^{2/3} - 2.81 (n-\frac{1}{4})^{2/3}$$
 (16)

or

$$(t_{n-z}) = 3.72 \times 10^{-2} (h-h_T) - 2.81 (n-1)^{2/3}$$
 (15a)

for f = 19.4 kH

and

$$(t_{n-2}) = 4.64 \times 10^{-2} (h-h_T) - 2.81 (n-1)^{2/3}$$
 (15b)

for f = 27 kHz

Similarly, for t_{n-2} , we can write:

$$(t_{n-2}) = 3.72 \times 10^{-2} (h-h_T) - 2.81 (n-1)^{2/3}$$
 (16a)

for f = 19.4 kHz

and

$$(t_{n-z}) = 4.64 \times 10^{-2} (h-h_T) - 2.81 (n-1)^{2/3}$$
 (16b)

for f = 27 kHz

Using equations (12), (15) and (16) in equation (11) we can develop the expression for HGF in a simplified form as follows:

HGF = h
$$\left(\frac{4\Pi^4}{6aC^2}\right)^{1/3} \left(n-\frac{1}{4}\right)^{-1/3} f^{2/3}$$

$$X = \left[A_{\frac{1}{4}} \left(\frac{2}{a} \left(\frac{\Pi a}{c}\right)^{2/3} f^{2/3} (h-h_T) - 2.83 \left(n-\frac{1}{4}\right)^{2/3}\right)\right]$$

$$X = \left[A_1 \left(\frac{2}{a} \left(\frac{\Pi a}{c}\right)^{2/3} f^{2/3} \left(h - h_R\right) - 2.83 \left(n - \frac{1}{4}\right)^{2/3}\right]\right]$$

or

HGF =
$$3.49 \times 10^{-2} h \left(n-\frac{1}{4}\right)^{-1/3}$$

$$X A_i(3.72 \times 10^{-2}(h-h_T) - 2.81 (n-1)^{2/3})$$

$$X = A (3.72 \times 10^{-2} (h-h_R) - 2.81 (n-1)^{2/3})$$

for
$$f = 19.4 \text{ kHz}$$
 (17)

and

$$HGF = 4.35 \times 10^{-2} h(n-1)^{-1/3} \times A_{1}(4.64 \times 10^{-2}(h-h_{T}) - 2.81(n-1)^{2/3})$$

$$\times A_{1}(4.64 \times 10^{-2} (h-h_{R}) - 2.81 (n-1)^{2/3})$$
(18)

for f = 27 Mz

Tabulated values of Airy functions, A, for different values of their arguments as obtained for various values of h_T and h_R , the heights of the transmitting and receiving antennas, are listed in Table I. The values have been obtained from reference (8). The appropriate values of $\alpha(dBMm)$, the attenuation constant, for different modes (mode 1 through 3), and for day and night time conditions are given in Table II. These values were obtained from reference (1).

From the values given in this table, it is found that the attenuation rate for the first order mode is less at the lower frequency during the daytime and increases during the nighttime. On the contrary, for the second and third order modes, the attenuation constant decreases during the nighttime. The trend is the same for the higher frequency (27 kHz).

Using the values of Airy functions for various augments as listed in Table II, the corresponding values of modified height gain factors for all the modes (mode 1 through 3) have been calculated for the condition when the receiving and transmitting antennas are at the same elevation. The controlling parameters are the transmitting/receiving antenna elevation, the propagation condition for the ionosphere, and the transmission frequency. The numerical values are given in Table II and shown in graphical form in Figures 1 and 2. The calculations have also been repeated for the condition when the receiving antenna is ground-based and the elevation of the transmitting antenna is increased above the ground. These tabulated values are given in Table III and the graphical forms have been represented in Figures 3 and 4. It should be emphasized at this point that these values of the modified height gain factors along with the corresponding values of the attenuation constant determine the values of the vertical component of the individual and multimode field strength as a function of distance and the resultant model interference pattern.

With these calculated values of α and HGF's, the amplitude of the vertical component of the field strength for any of the modes (mode 1 through 3), for any frequency (either 19.4 or 27 kHz) and for any propagation condition of the ionosphere (h=70 km for the daytime condition and h=90 km for the nighttime condition) can be computed by using the Equations 7 through 10.

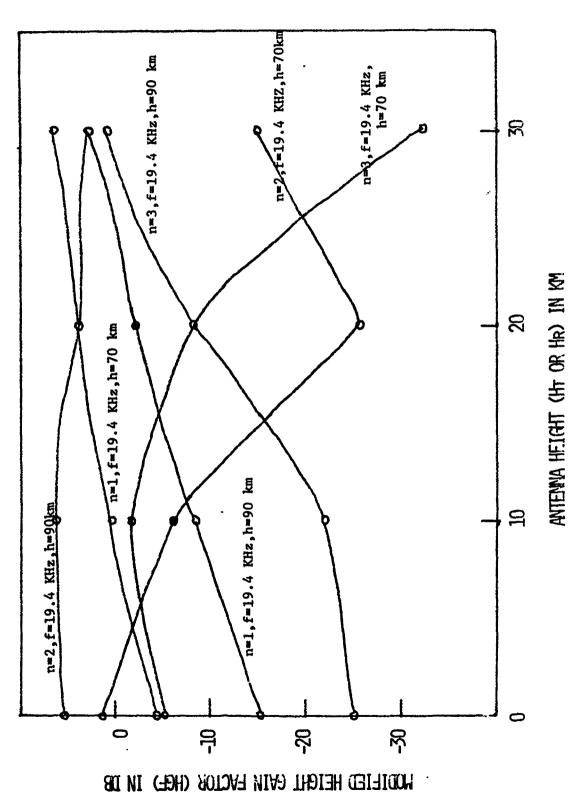
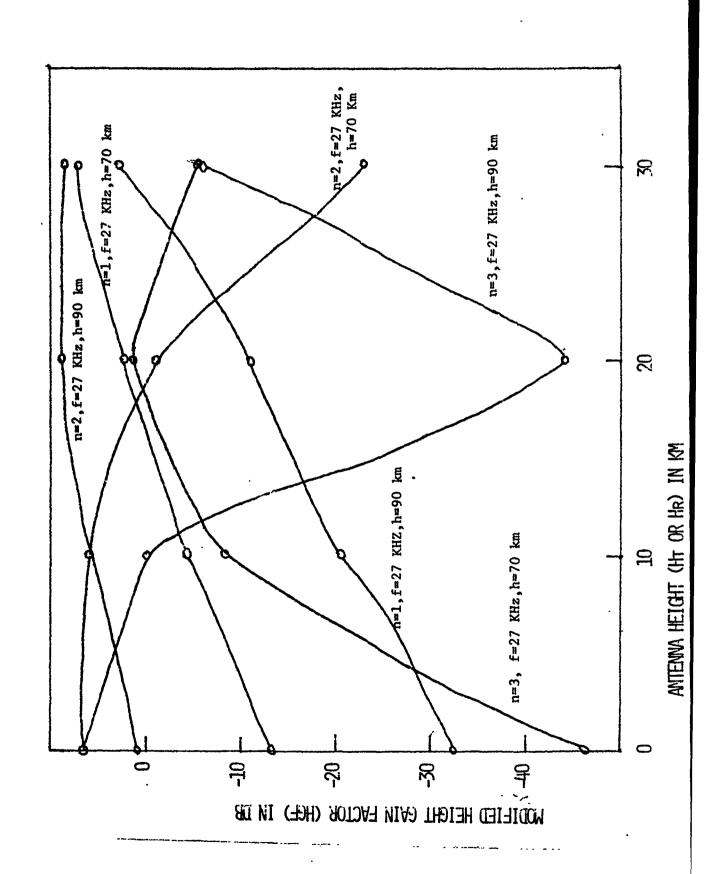
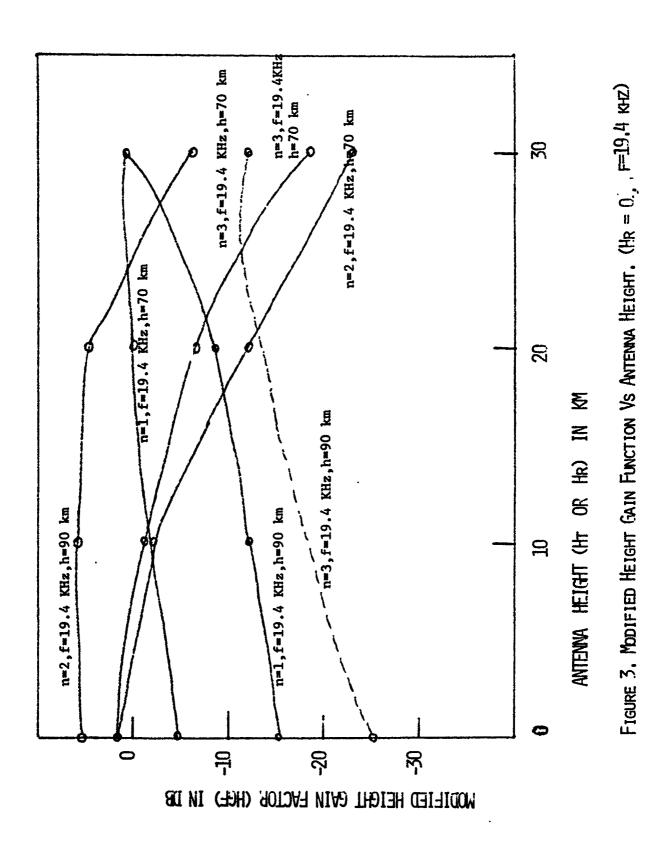


Figure 1. Modified Height Gain Factor Vs Antenna Height ($H\tau=HR$, $F=19.4~{\rm kHz})$

FIGURE 2. MODIFIED HEIGHT GAIN FUNCTION VS ANTENNA HEIGHT (H_{T} = H_{R} , F = 27 KHZ)

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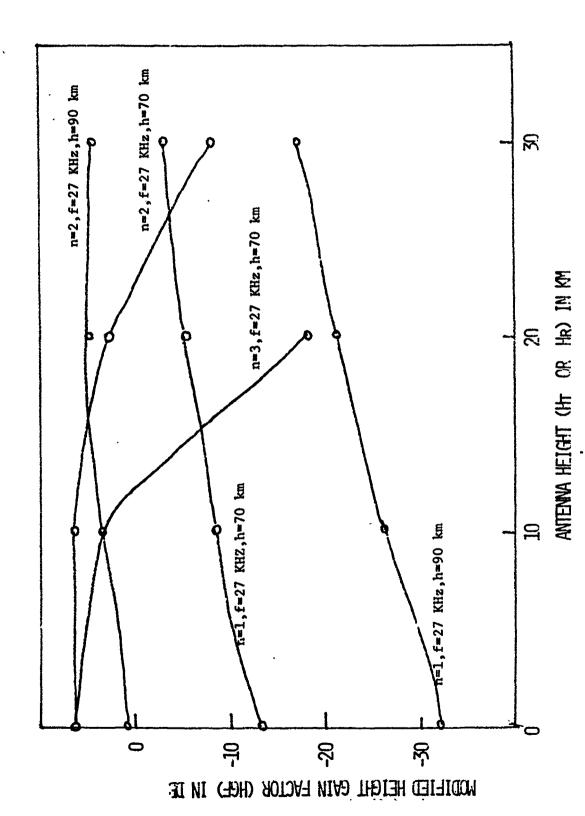


FIGURE 4, MODIFIED HEIGHT GAIN FACTOR VS ANTENNA HEIGHT (HR=0, F=27 KHZ)

TABLE I. VALUE OF J, AN AUXILIARY VARIABLE

f (kHz)	<u>h (km)</u>	<u>n</u>	α (dB/Mm)	$\sigma = 4.83 \times 10^{-3} \text{ h f}^{2/3} \left(n - \frac{1}{4}\right)^{-1/3}$
19.4	70	1 2 3	1.56 6.56 15.19	7.2193 5.443 4.682
	90	1 2 3	1.62 3.20 7.80	9.282 6.998 6.019
27	70	1 2 3	2.00 4.54 10.54	10.047 7.575 6.516
	90	1 2 3	2.48 2.30 5.56	12.918 9.740 8.377

NADC-83062-40 TABLE II. VALUES OF MODIFIED HEIGHT GAIN FACTORS ($h_T = h_R$)

f (kHz)	h (km)	h _T (km)	n	t _n -z	Ai (tn-z)	HGF (dB)
19.4	70	0	1	0.2844	0.2837	- 4.72
				-1.4767	0.4643	1.39
			2 3	-2.9116	-0.3419	- 5.24
		10	1	-0.0876	0.3783	0.28
			2 3	-1.8487	0.3008	- 6.15
			3	-3.2836	-0.4172	- 1.78
		20	1	-0.4596	0.4674	3.96
			2	-2.2207	0.0962	-25.96
			3	-3.6556	-0.282	- 8.58
		30	1	-0.8316	0.5266	6.03
			2	-2.5927	-0.1785	-15.22
			3	-4.0276	-0.0703	-32.71
	90	0	1	1.0284	0.1353	-15.4
			2	-0.7327	0.5152	5.38
			3	-2.1676	0.096	-25.12
		10	1	0.6564	0.1973	- 8.84
			2	-1.1047	0.5338	6.0
			3	-3.5396	-0.1123	-22.39
		20	1	0.2844	0.2834	- 2.55
			2	-1.4767	0.4643	3.57
			3	-2.9116	-0.3419	- 3.05
		30	1	-0.0876	0.3783	2.47
			2	-1.8487	0.2868	- 4.80
			3	-3.2836	0.4172	0.40
27	70	0	1	0.9284	0.1468	-13.29
			2	-0.8327	0.5266	6.45
			3	-2.2676	0.0267	-46.66
		10	1	0.4644	0.2408	- 4.69
			2	-1.2967	0.5123	5.97
			3	-2.7316	-0.2400	- 8.51
		20	1	3.9611×10^{-4}	0.3550	2.05
			2	-1.7607	0.3408	- 1.14
			3	-3.1956	-0.4175	1.11
		30	1	-0.4636	0.4674	6.83
			2	-2.2247	0.0961	-23.10
			3	-3.6596	-0.2820	- 5.71
•	90	0	1	1.8564	0.0431	-32.40
			2	0.0953	0.3318	0.61
			3	-1.3396	0.5123	6.84

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TABLE II (CONTINUED)

f (.len;z)	h (km)	h _T (km)	<u>:n</u>	t _n =z	Ai (tn-z)	HGF (dB)
27	90	10	1	1.3924	0.0834	-20.93
			2	-0.3687	0.4474	5.80
			3	-1.8036	0.3408	- 0.24
		20	1	0.9284	0.1468	-11.11
			2	-0.8327	0.5266	8.63
			3	-2.2676	0.0267	-44.48
		30	. 1	0.4644	0.2408	2.51
			2	-1.2967	0.5123	8.15
			3	-2.7316	-0.2400	- 6.33

. NADC-83062-40 TABLE III.VALUES OF MODIFIED HEIGHT GAIN FACTORS (${}^{h}_{R} \neq {}^{h}_{T}$)

f ('kH.z)	'h (km.)	ha (km)	hr (km)	<u>'n</u>	HGF (B)
19.4	70	0	0	1 2 3	- 4.72 1.39 - 5.24
			10	1 2 3	- 2.22 - 2.38 - 3.51
			20	1 2 3	- 0.38 -12.27 - 6.91
			30	1 2 3	0.66 -23.42 -18.98
	90	0	0	1 2 3	-15.40 5.38 -25.12
			10	1 2 3	-12.12 5.69
			20	1 2 3	- 8.97 4.48
			30	1 2 3	- 6.47 0.29 -12.35
27	70	0	0	1 2 3	-13.29 6.45 -46.66
			10	1 2 3	- 8.99 6.21
			20	1 2 3	- 5.62 2.67
			30	1 2 3	- 3.23 - 8.33

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TABLE III (CONTINUED)

f (kHz)	h (km)	h _R (km)	h _T (km)	<u>n</u>	HGF (dB)
· 27	90	0	0	1	-32.40
				2	0.61
				3	6.84
			10	1	-26.66
				2	3.20
	•			3	3.30
			20	1 .	-21.75
				2	4.62
				3	-18.82
			30	1	-17.45
				2	4.38
				2 3	

MULTIMODE FIELD STRENGTHS

At long distances the field strengths due to second and third order mode are equally important. Hence, total field strengths at large distances are the vector sum of individual mode field components. To calculate such resultant field strength, a knowledge of the phase of the individual mode at a certain distance is essential.

The phase of the vertical component of the nth mode (n = 1, or 2 or 3) field strength with respect to the transmitting antenna current is given by equation (4) as

$$\phi_{E_{z,n}} = \frac{\pi}{4} - \omega D / v_{p,n} + \phi_{\Lambda_n}$$

$$= \frac{\pi}{4} - \frac{2\pi Dc}{\lambda v_{p,n}} + \phi_{\Lambda_n}$$

where c is the speed of light in free space and the other terms have their previously defined meanings. The values of $c/v_{p,n}$ and $\phi_{\Lambda n}$, as obtained from reference (1), are given in Table III. With the values of $c/v_{p,n}$ and $\phi_{\Lambda n}$, values of $\phi_{E_{Z,n}}$ for any mode, the frequency, distance and ionospheric conditions can be calculated from the equation cited above. The values of $\phi_{E_{Z,n}}$ thus calculated for different modes and frequencies are listed in Table IV. From this table, it is also found that the difference in phase between first and second order modes for f = 19.4 kHz is

$$\Delta \phi_{1,2}$$
 = 5.01 radian/Mm during daytime (h =70 km)
= 3.20 radian/Mm during nighttime (h =90 km)

and for f = 27 kHz

$$\Delta \phi_{1,2} = 4.22 \text{ radian/Mm} \text{ during daytime (h=70 km)}$$

2.33 radian/Mm during nighttime (h=90 km)

Furthermore, with known values of $E_{z,n}$, the amplitude of the vertical component of the multimode field strength is calculated as follows:

$$E = ((E_{z1} \cos \phi_{E_{z1}} + E_{z2} \cos \phi_{E_{z2}} + E_{z3} \cos \phi_{E_{z3}})^{2}$$

$$(E_{z1} \sin \phi_{E_{z1}} + E_{z2} \sin \phi_{E_{z2}} + E_{z3} \sin \phi_{E_{z3}})^{2})^{1/2}$$

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TABLE	IV.	- VALUES	FOR	ϕ^{0}_{n} ,	v_{pn}	AND	φn
					С		•

f (kHz)	h (km)	n.	ϕ°_{n}	$\frac{v_{pn}}{c}$	<u>φn</u>
19.4	70	1 2 3	7.50 1.40 0.78	0.9984 1.011 1.042	0.916-0.407D 0.809-0.402D 0.799-0.390D
	90	1 2 3	13.12 1.82 0.66	0.996 1.0038 1.02	1.014-G.408D 0.817-0.405D 0.797-0.398D
27	70	1 2 3	14.2 1.6 0.42	0.997 1.0035 1.017	1.033-0.567D 0.813-0.563D 0.792-0.556D
	90	1 2 3	25.5 6.5 -0.03	0.9944 0.9994 1.0065	1.23-0.568D 0.898-0.566D 0.785-0.562D

φ is calculated from

$$\phi = \frac{\pi}{4} - \frac{\omega D}{v} + \phi_n \qquad r_{ad}$$

$$= \frac{\pi}{4} - \frac{2}{\lambda} D \frac{c}{v_{pn}} + \phi_n$$

where λ , D in k_{\perp} , $\lambda \sim \frac{300}{f}$, f in k_{\parallel}^{Hz} .

and the corresponding phase is calculated from

$$\phi = \tan^{-1} \left(\frac{E_{z1} \sin \phi_{Ez1} + E_{z2} \sin \phi_{Ez2} + E_{z3} \sin \phi_{Ez3}}{E_{z1} \cos \phi_{Ez1} + E_{z2} \cos \phi_{Ez2} + E_{z3} \cos \phi_{Ez3}} \right)$$
(20)

DISCUSSION

A microcomputer (Tektronix 4054 with accompanying Hard Copy Unit 4631) was used for all the computational works reported in this report. A computer program written in BASIC and provided in Appendix A has been used to compute the values of Ezn, the vertical component of the individual mode electric field strength as a function of distance based on any of the Equations 7 through 10. The sample program shown in Appendix B was used for the computation of the values of Ez, the multimode field strength based on Equation 19 and provides the output in graphical and tabular forms. These graphical plots have been represented in Figures 5 through 20. For all these plots, the transmitting and receiving antennas are assumed to be at the same elevation and the controlling parameters use either the frequency, the height of the ionospheric reflection point, or the elevation of the transmitting/receiving antennas. Distances are varied up to 3000 km and the elevation of the transmitting/receiving antennas up to 30 km in order to conform to the plans of the experiment. Figures 21 through 32 illustrate similar results when the receiving antenna is ground-based and the transmitting antenna elevation is varied up to 30 km in steps of 10 km.

The signature of strong interference phenomenon due to individual modes on the modesum field strength is apparent and it is indicative of the fact that at many locations, although the individual mode field strength is strong, the modesum field strength is relatively weak due to modal interference. At low frequency (19.4 kHz) and during daytime, the interference pattern begins to appear at short distances. For the ground-based transmitting and receiving antennas, the modesum field strength curve displays strong interference pattern with bit nulls occurring at 600 km and 2000 km as can be seen in Figure 5. The interference phenomenon becomes weaker as the antenna elevation is increased and at an antenna height around 30 km it disappears completely as is seen in Figure 8. This variation of interference phenomenon can be attributed to the fact that at higher altitude, the height gain factor for the first order mode increases and for the second and third order mode it decreases. Furthermore, the attenuation constant for the first order mode is very low (1.5 dB/Mm) whereas for the second and third order modes attenuation constant is fairly high (5.6 dB/Mm and 15.2 dB/Mm). All these factors combine together to make the field strength curve for the first order mode predominant at large distances thereby making the interference pattern very weak.

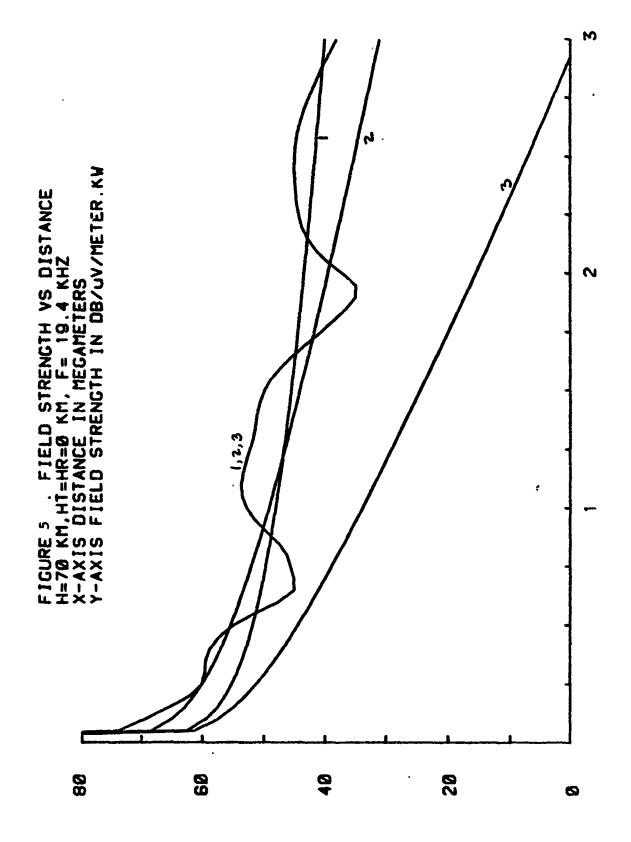
Interference patterns during nighttime are illustrated in Figures 9 through 12. It is seen in these figures that at nighttime, the interference phenomenon is very weak when the antenna elevation is low and gets stronger as the elevation is increased and is strongest when the antenna height is 30 km. A prominent null occurs at 1000 km as is seen in Figure 12. Referring to Figure 1, it is found that at nighttime the attenuation rate for the first order more is higher than during the daytime whereas for the second and third order modes, this rate is lower. The height gain factor for the second order mode does not fall off fast with antenna elevation and for the third order mode it rather increases. The net effect of these factors is to make the second and third order mode field strengths very strong and results in a strong interference pattern for the multimode field strength curve.

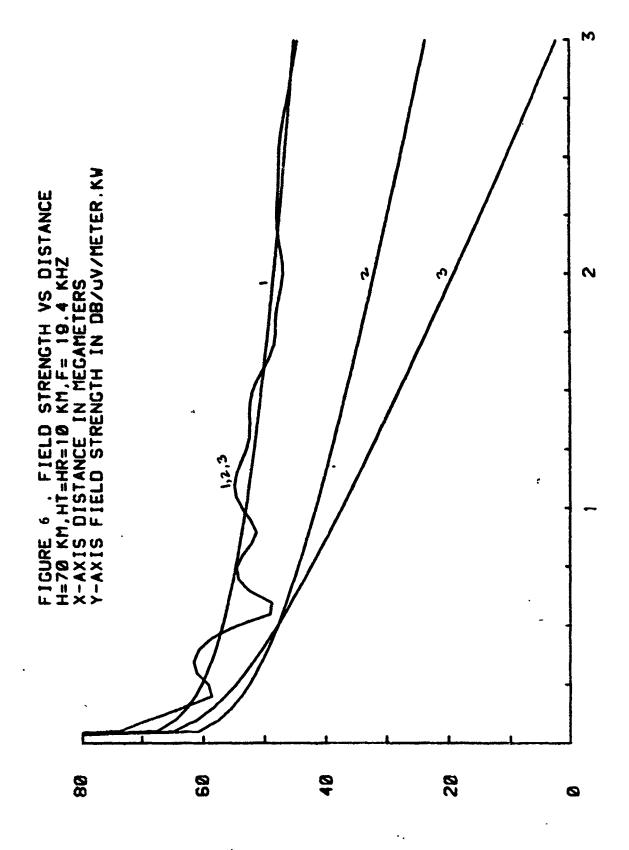
At the higher frequency (27 kHz) and during the daytime propagation condition, a reverse condition for the interference is displayed by the multimode field strength curve as is shown in Figures 13 through 16. At lower antenna elevation, the second order mode curve is weak and the third order mode curve is weakest making the multimode interference phenomenon less prominent. As the antenna elevation increases, second and third order mode curves become stronger and a strong interference pattern is observed with a deep null occurring at 850 km when the transmitting and receiving antenna elevation is 20 km as is illustrated in Figure 15. As Figure 16 shows, above this height the interference phenomenon becomes weak.

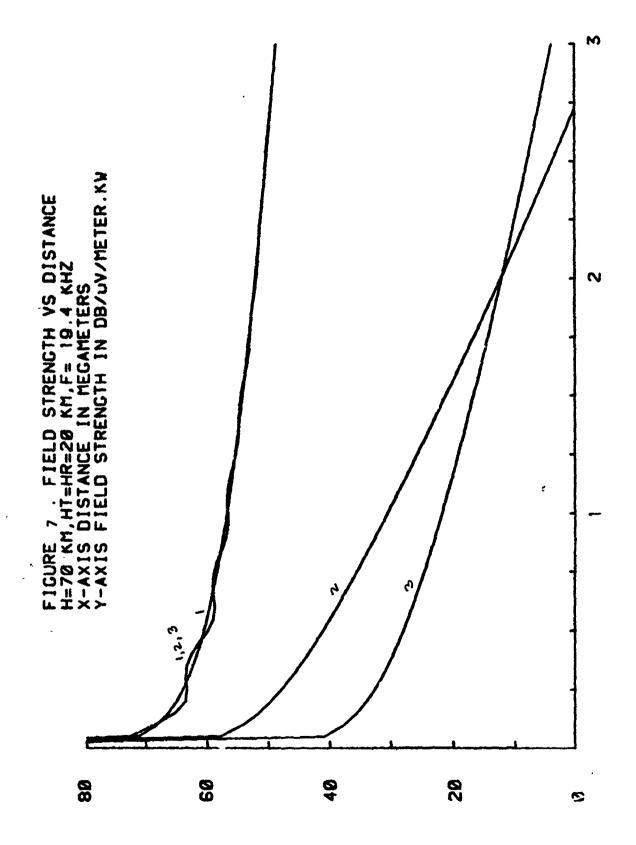
Figures 17 through 20 show the interference pattern for the higher frequency (27 kHz) during the nighttime condition. For the ground-based antennas strong modal interference accompanied by two deep nulls occurs. The intensity of interference decreases as the transmitting/receiving antenna elevation is increased.

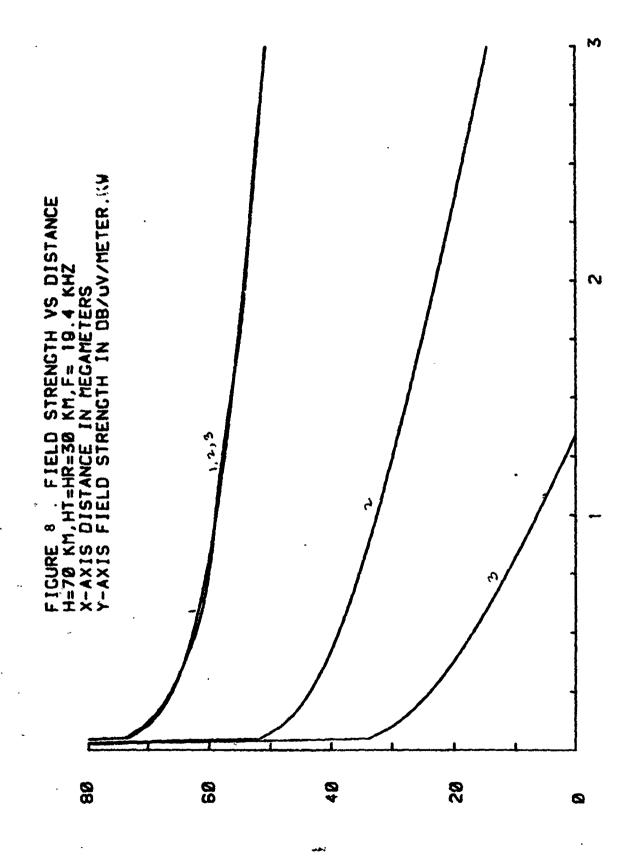
The interference phenomenon is also present in the multimode field strength curve as a function of distance both during the daytime and nighttime when the receiving antenna is ground-based and the transmitting antenna is elevated. The interference pattern changes from weak to strong and vice versa when this elevation is increased up to 30 km. As an example for the lower frequency, it is seen in Figure 21 that at closer distances and at lower antenna elevations, the height gain factors for the second and third order mode are fairly high making their corresponding field strengths strong. Interference phenomenon begins to appear at short distances, and a strong null occurs at 600 km. At higher transmitting antenna elevation, the interference pattern fades away as the second and third order mode fields become less predominant. At nighttime, the interference phenomenon is not so strong.

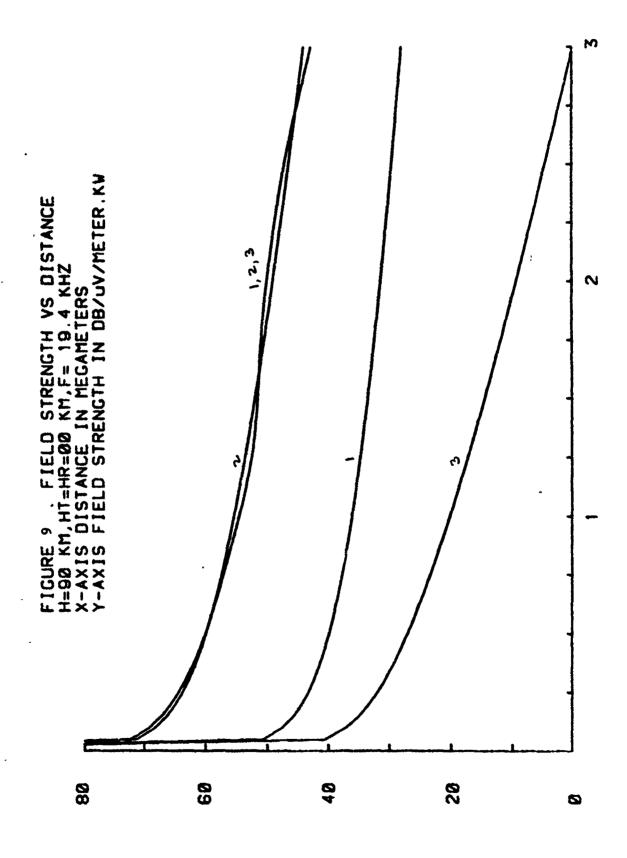
At higher frequency and during the daytime condition, the interference phenomenon is strong for the intermediate elevation of the transmitting antenna, the receiving antenna being ground-based and is characterized by the occurence of two strong nulls, one at 800 km and the other at 2400 km as are shown in Figure 24. During nighttime, a strong interference phenomenon occurs only when the transmitting antenna elevation is around 10 km. At higher elevations, the second and third order mode become less significant and the interference pattern disappears - making the multimode field strength curve equivalent to the first order mode field strength curve.

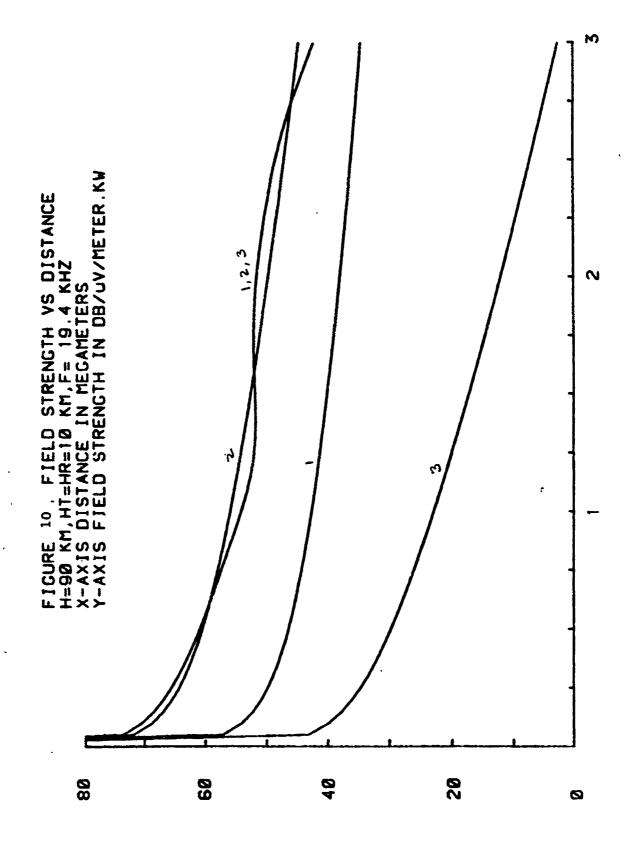


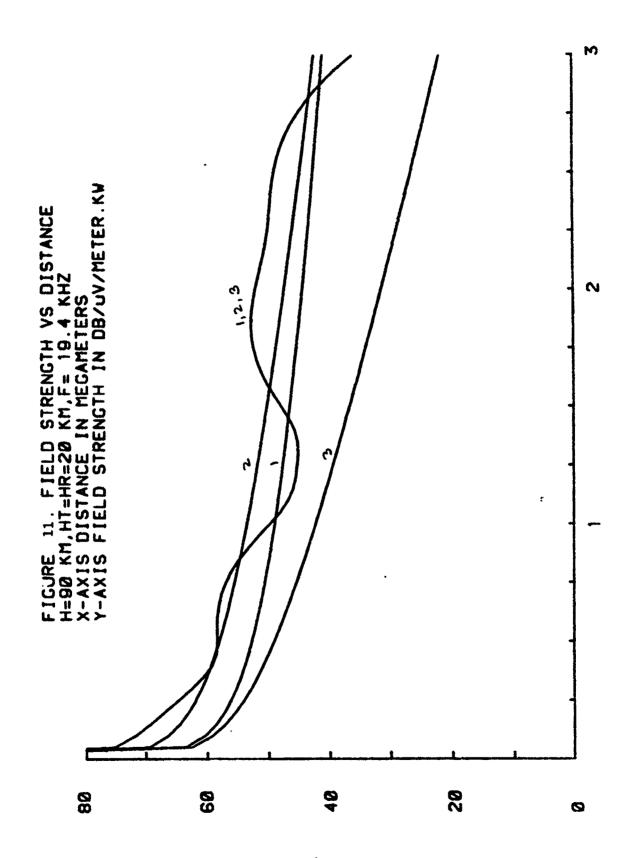


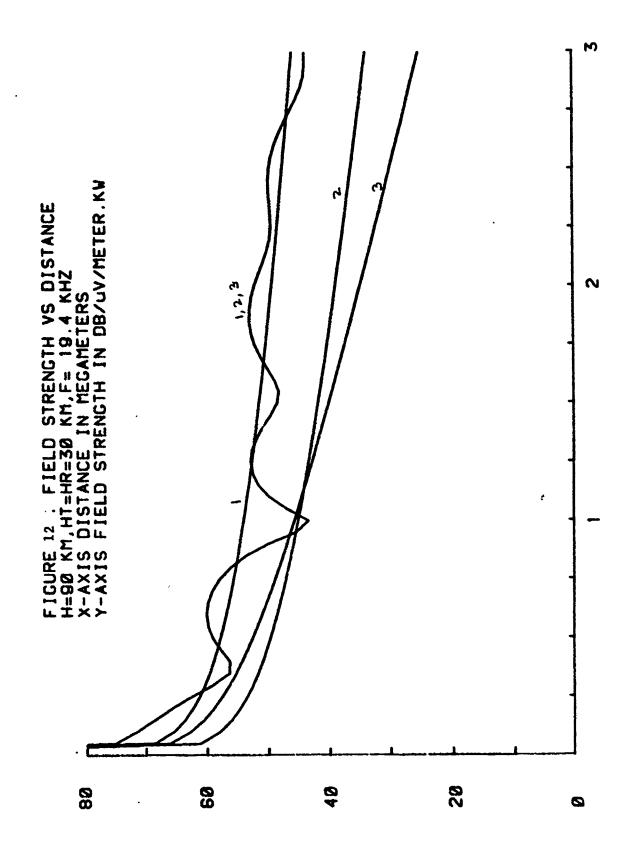


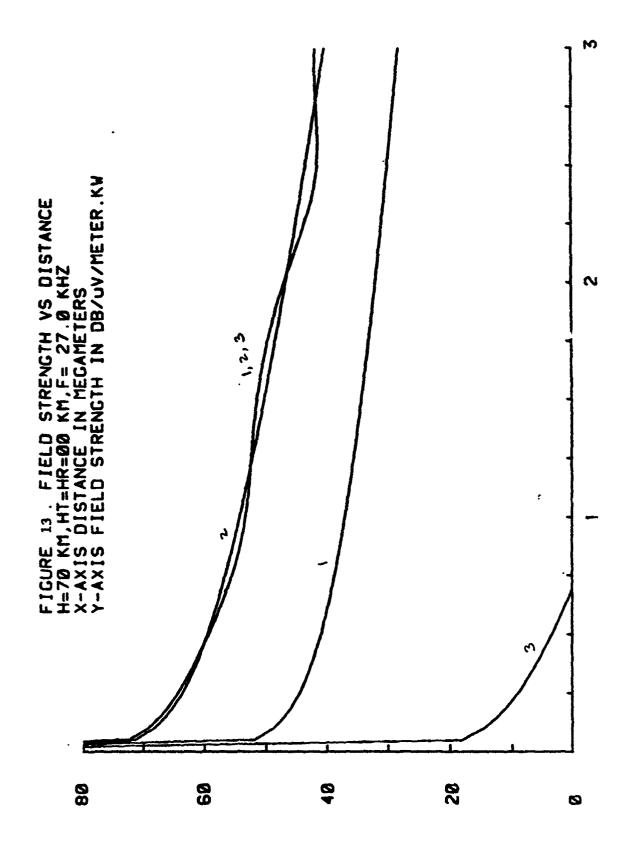


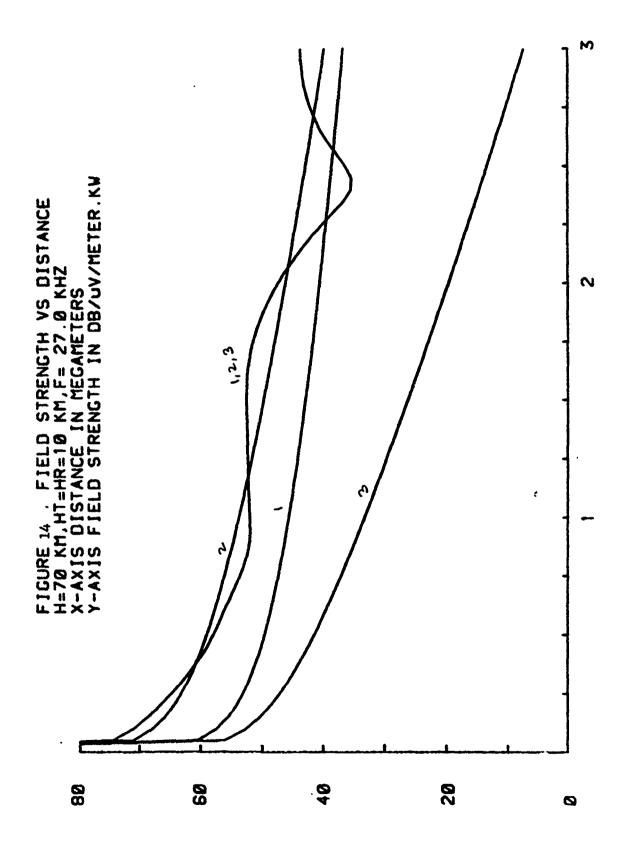


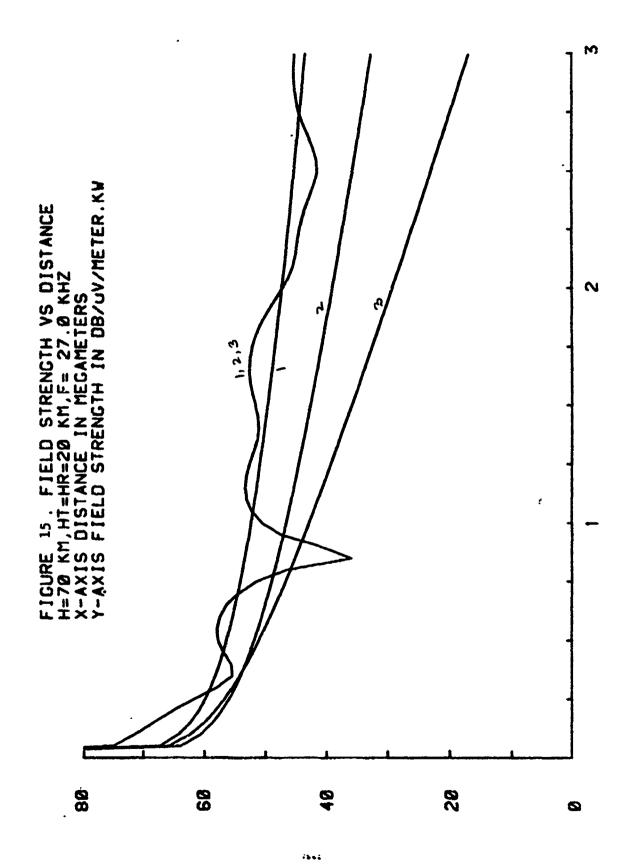


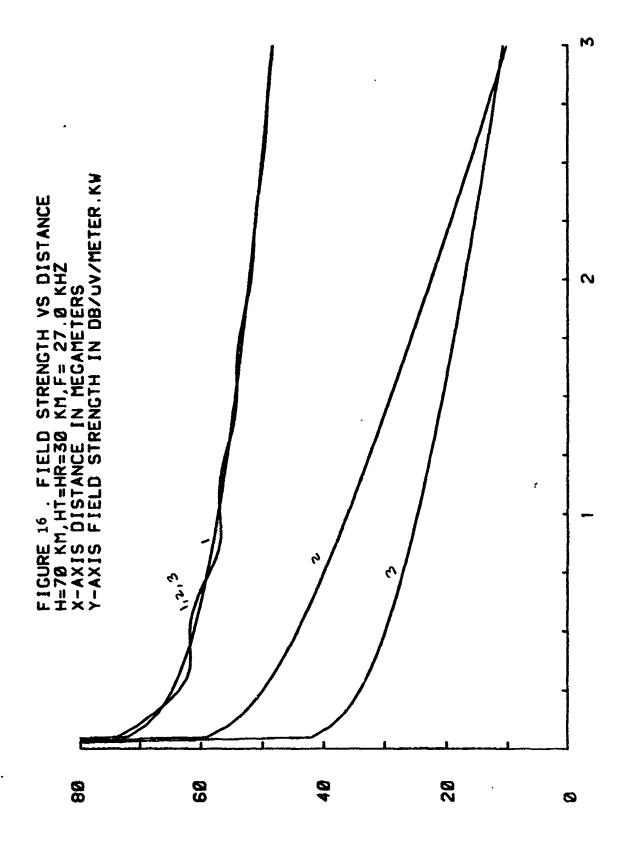


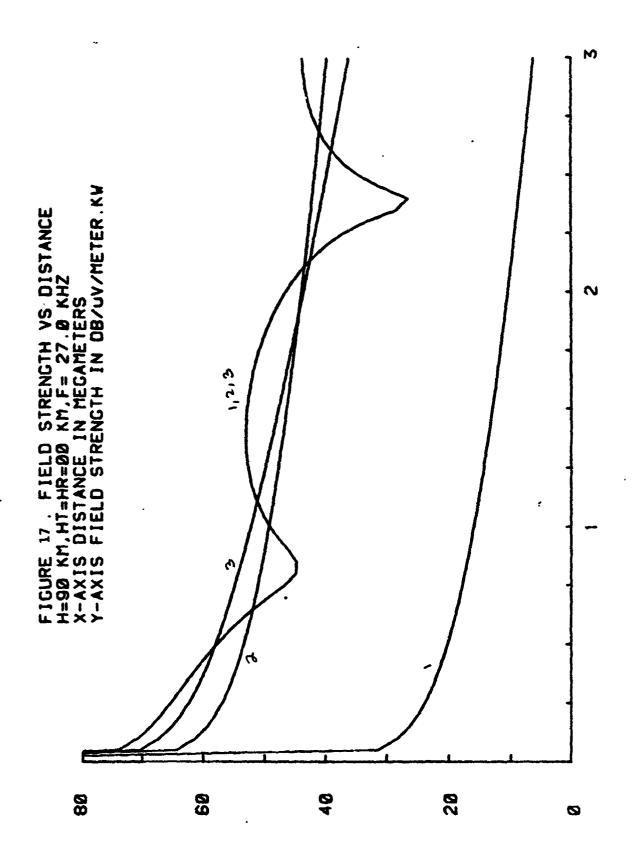


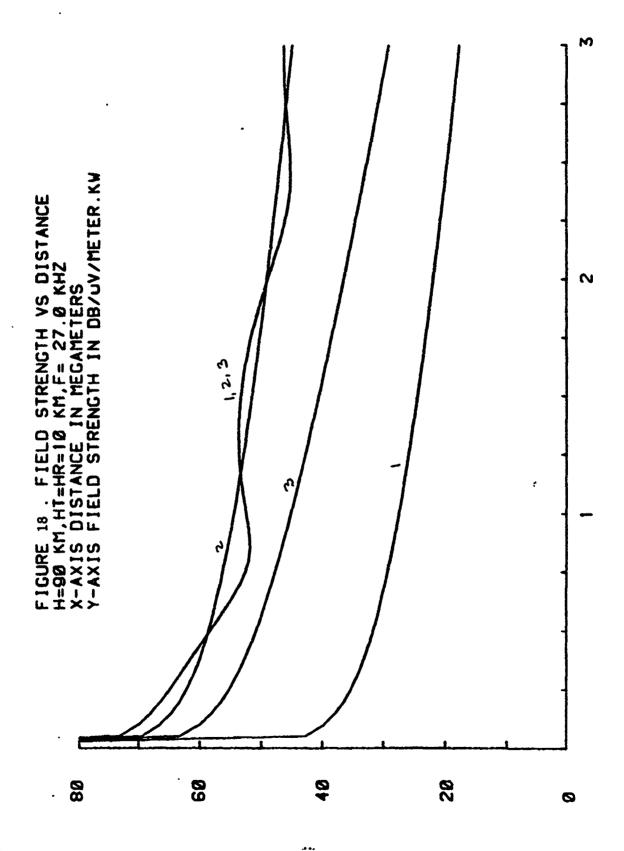


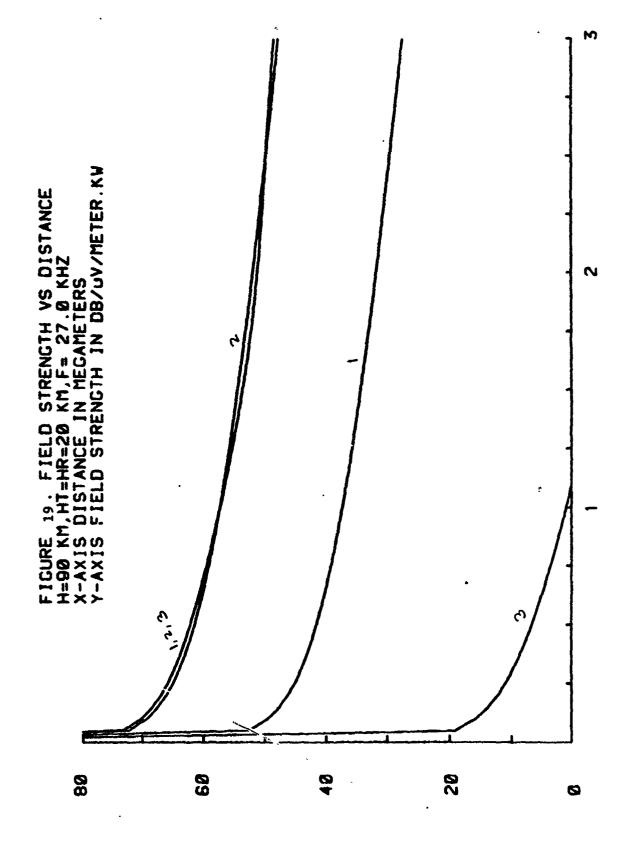


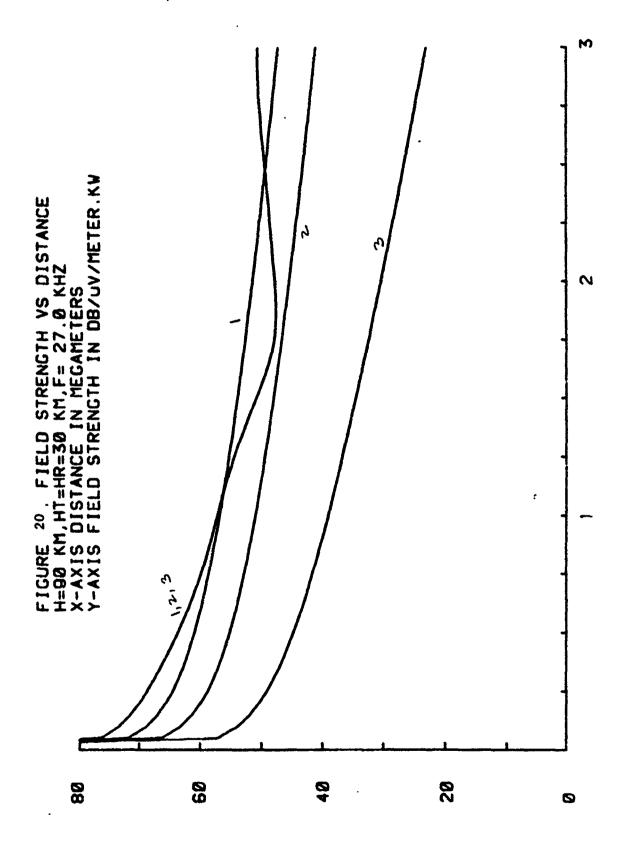


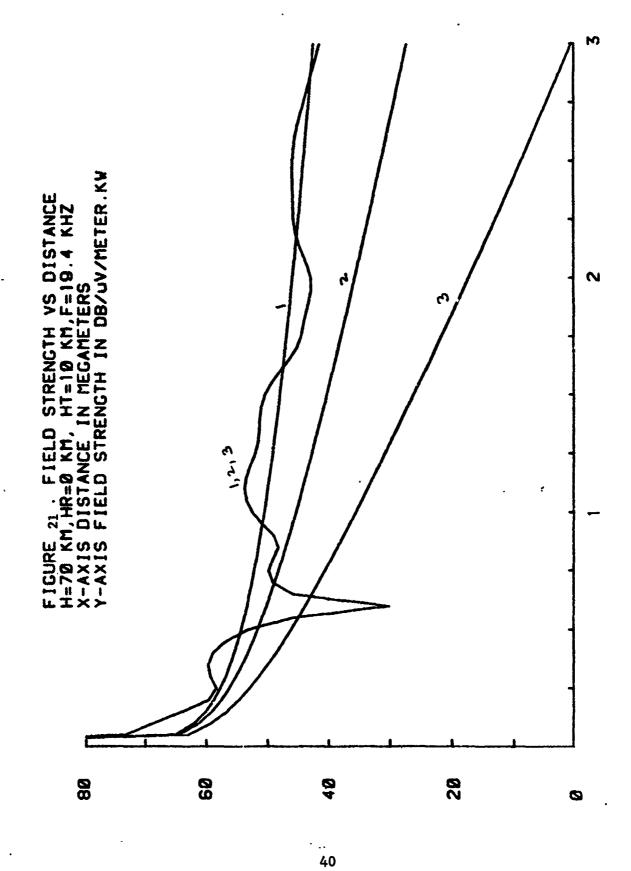


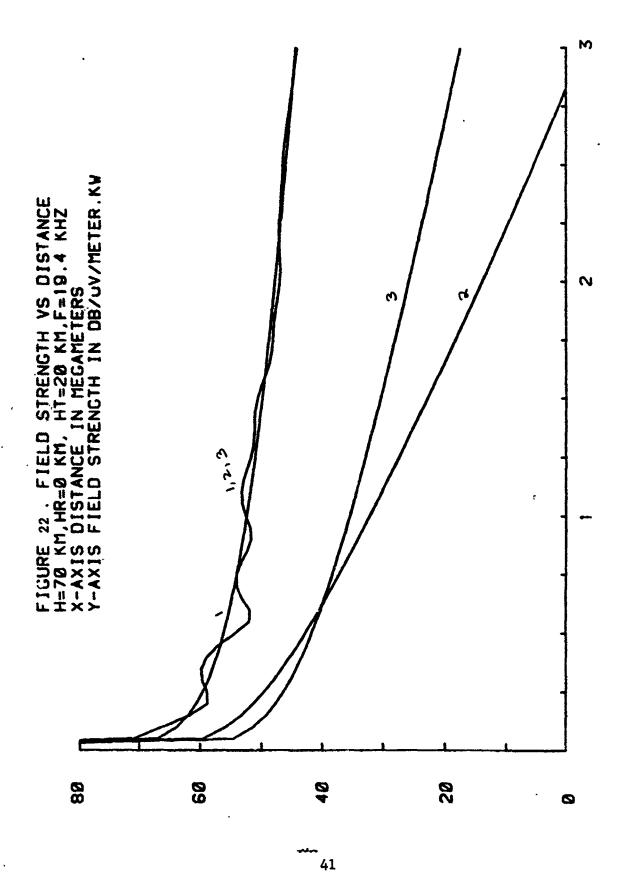


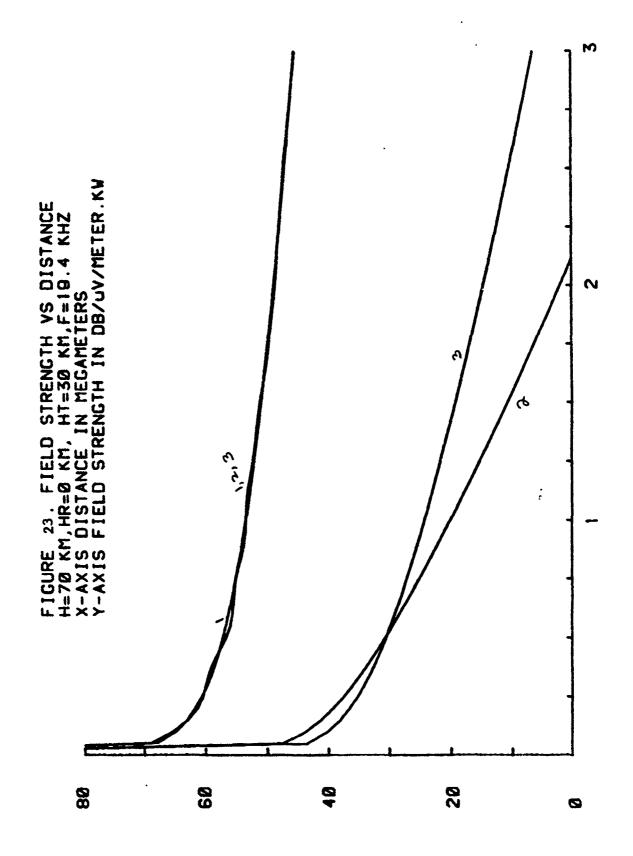


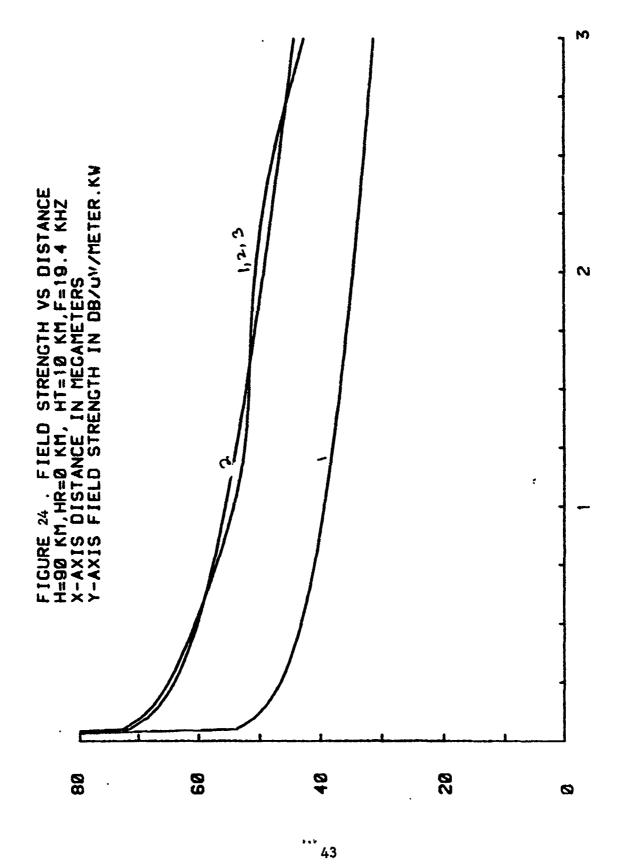


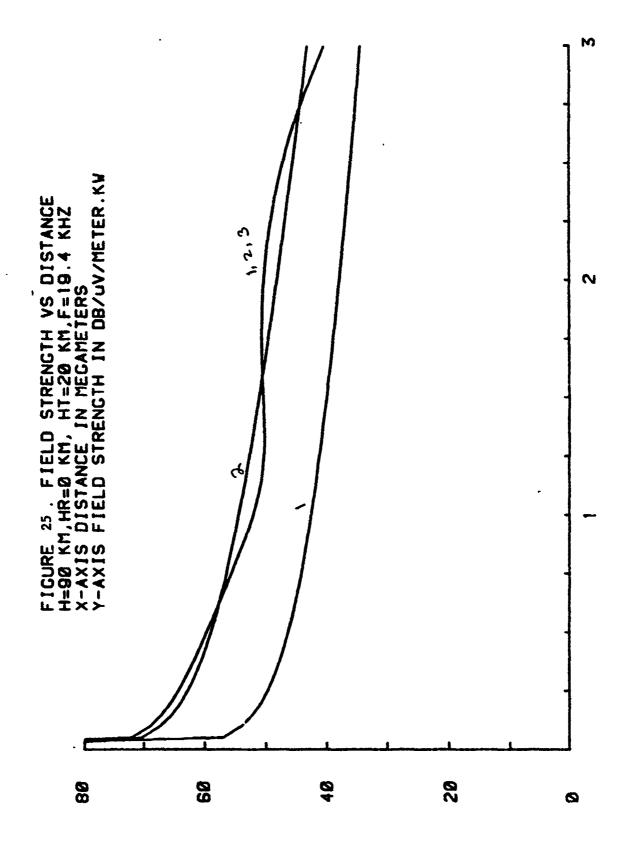


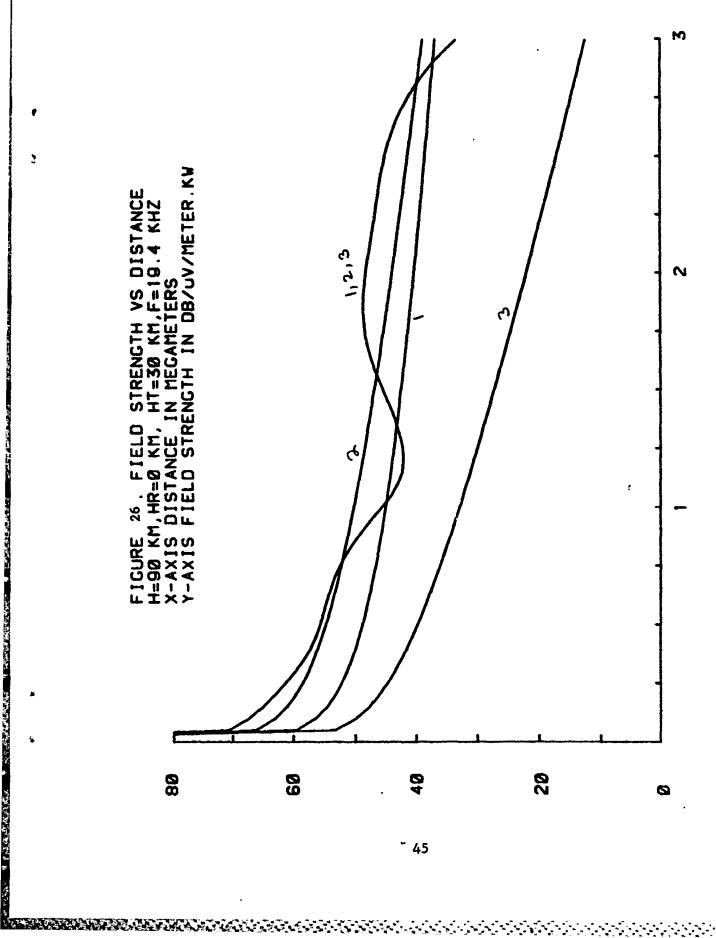


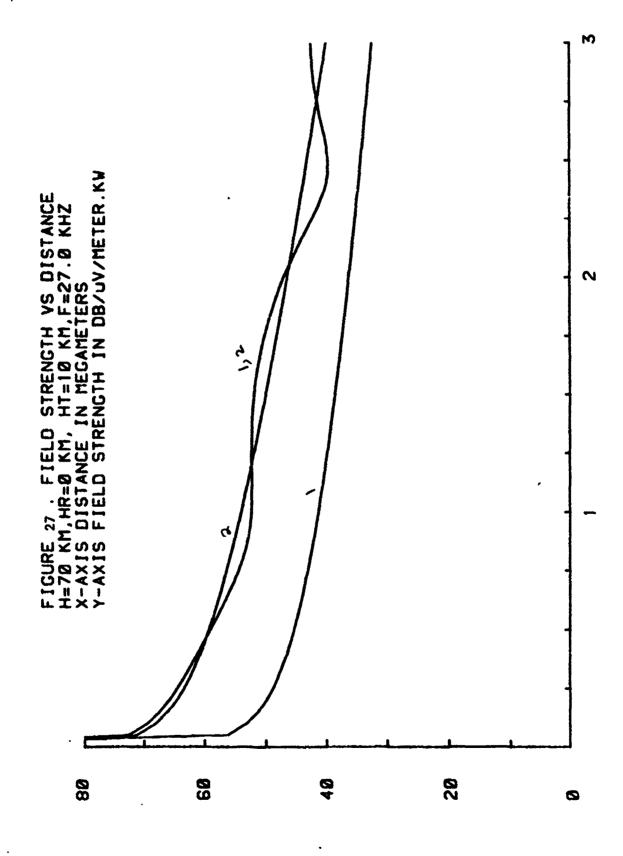


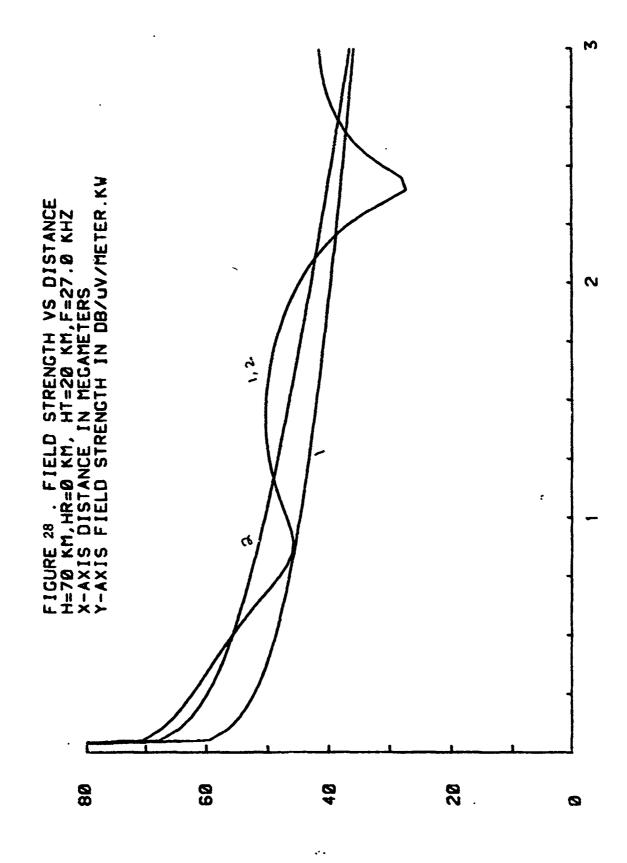


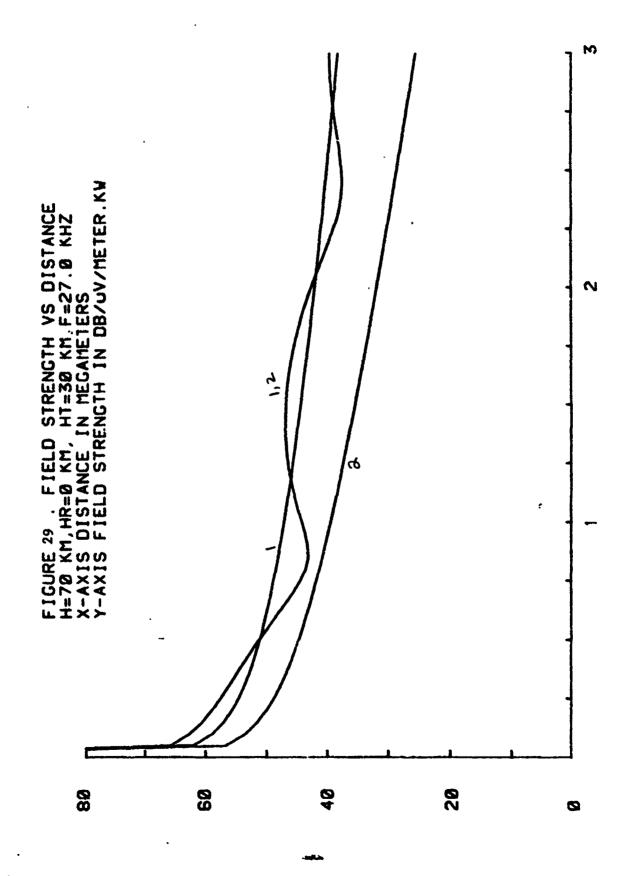


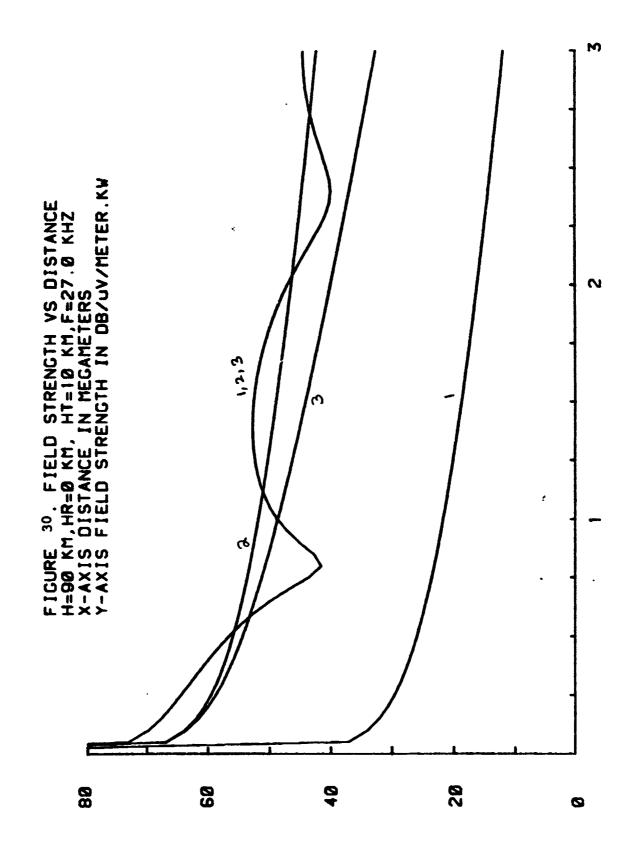


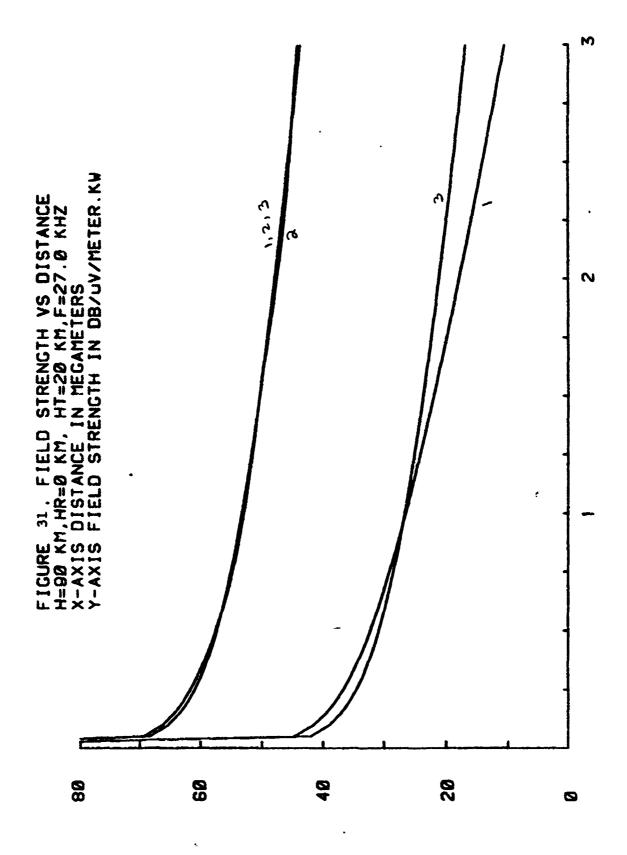


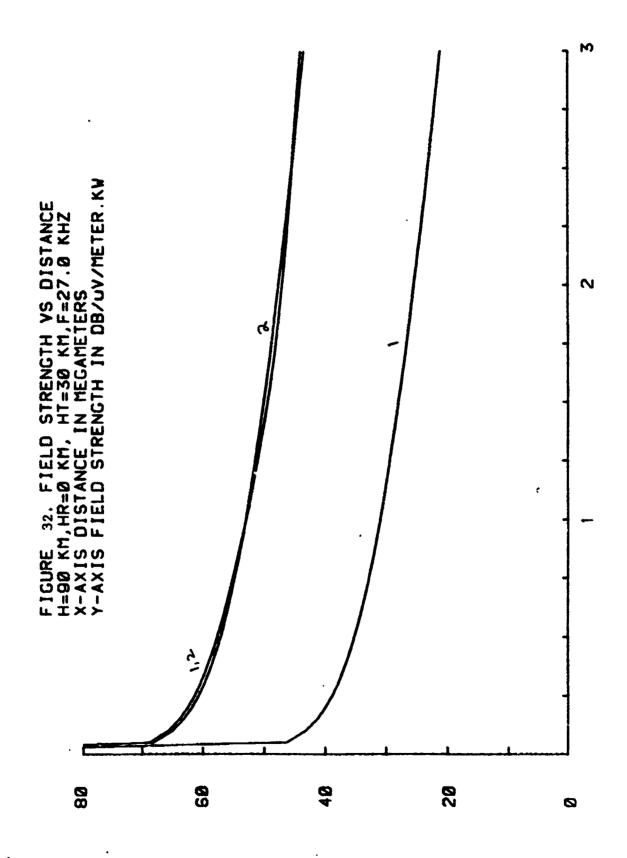












INFLUENCE OF GEOMAGNETIC FIELD ON MODESUM

AND INDIVIDUAL MODE FIELD STRENGTHS

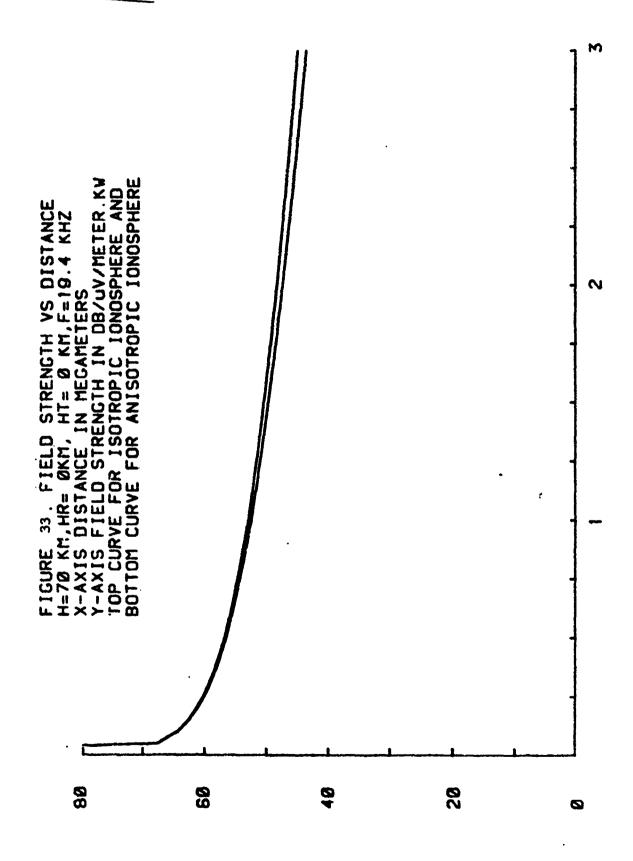
In all the previous calculations, it has been tacitly assumed that the ionosphere is isotropic and is characterized by an exponential profile for electronic density and collisional frequency variations. This assumption neglects the presence of the earth's geomagnetic field. When the presence of this field is taken into consideration, the ionosphere becomes anisotropic and the characteristics of VLF radio propagation change, though in most cases these changes are very small. The presence of the geomagnetic field produces changes in the values of individual mode and modesum field strengths through the changes in the values of phase velocity, phase of the excitation factor and attenuation constant, especially when the propagation takes place along the East-to-West or West-to-East directions. It has been experimentally observed that the field strengths for the East-to-West propagation is weaker than for the West-to-East propagation, in clear violation of the principle of reciprocity for waveguide mode of propagation (9).

The plans for the balloon-to-balloon-borne communication experiment calls for one of the links to have East-to-West propagation (Kwajalein, the transmitting site to Guam, the receiving site). In order to make an estimate of the influence of the geomagnetic field on this communication link, some of the calculations - especially for the predominant mode (the first order mode) - have been repeated for an anisotropic ionosphere. The calculations are applicable for a field station with a dip angle of 7° , which is the dip angle midway between Kwajalein-Guam link (data courtesy of the Geomagnetic Division, World Data Center A, Boulder, Colorado). The values of the various parameters for an anisotropic ionosphere are listed in Table V. Field strengths for the first order mode at 19.5 kHz as a function of distance for both the isotropic and anisotropic ionosphere have been presented in graphical forms in Figures 33 through 36. Results for the daytime condition with the transmitting and receiving antennas having the same elevations (either groundbased or 30 km) are shown in Figures 33 and 34. Figures 35 and 36 represent similar results for the nighttime condition. In all these figures, the top curve is for the isotropic ionosphere and the bottom curve for the anisotropic ionosphere. The corresponding numerical values of the field strengths and the differences of the field strengths are listed in tabular forms in Table VI through XVII. In all these analyses, it is found that the field strength differences occur with a maximum value of about 3 dB at a distance Thus, the anisotropy of the ionosphere, though important, is not very significant.

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TABLE V. PARAMETERS FOR ISOTROPIC AND ANISOTROPIC IONOSPHERE

h (Km)	£	φΛ1 (E→W)	фЛ1 +	α (dB/Mm) (E+W)	α (dB/Mm) · (N+S) ·	$\frac{v_{p1}}{(E+W)} - 1$	(N+S)
	H	11.1	10	2.8	2.5	-0.11	-0.11
		20	18	2.75	2.45	-0.35	-0.35
		20.1	19	3.75	3.4	-0.22	-0.22
		47	41	5.2	4.6	-0.52	-0.52



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VALUES OF FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ISOTROPIC IONOSPHERE ($h_{\rm T}$ = $h_{\rm R}$ = 0, h = 70 km, f = 19.4 kHz) TABLE VI.

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TABLE VII. VALUES OF FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ANISOTROPIC IONOSPHERE ($h_{\rm T}$ = $h_{\rm R}$ = 0, h = 70 km , f = 19.4 kHz)

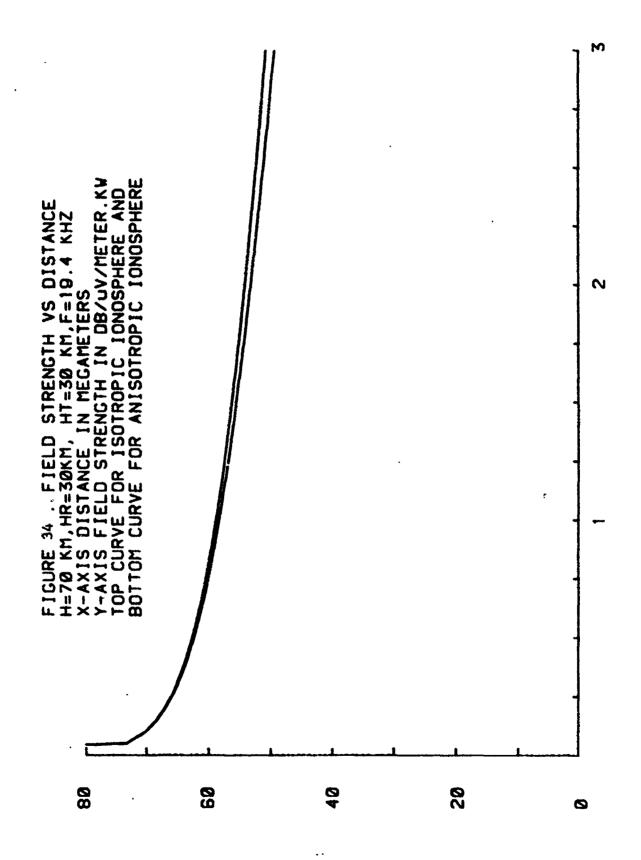
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TABLE VIII.



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VALUES OF FIRST ORDER MODE FIELD STRENGTH VS DISANCE FOR ISOTROPIC IONOSPHERE ($h_{\rm T}$ = $h_{\rm R}$ = 30 km, h = 70 km, f= 19.4 kHz) IX. TABLE

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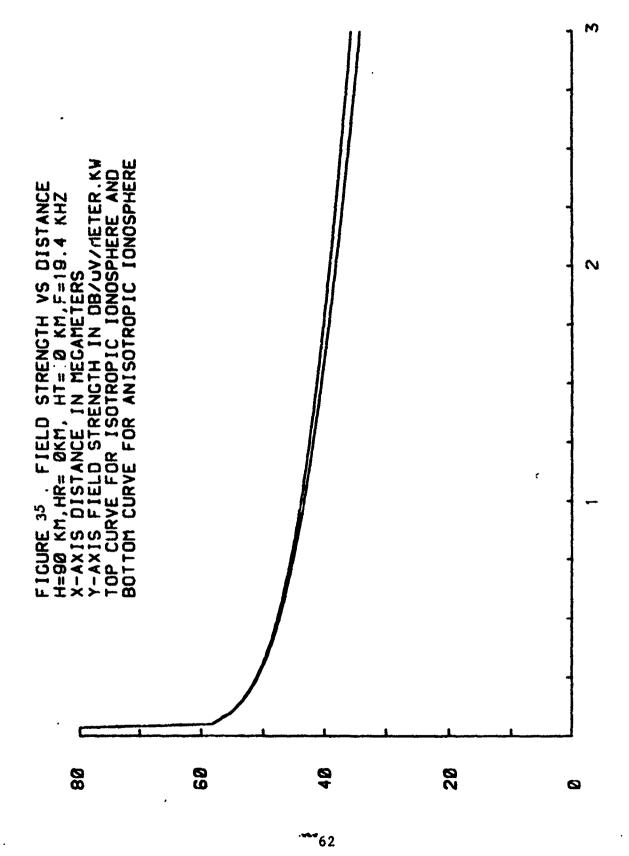
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VALUES OF FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ANISOTROPIC IONOSPHERE ($h_{\rm T}$ = $h_{\rm R}$ = 30 km, h = 70 km, f = 19.4 kHz) TABLE X.

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VALUES OF THE DIFFERENCE BETWEEN FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ISOTROPIC AND ANISOTROPIC IONOSPHERE . (h_T = h_R = 30 km, h = 70 km, f = 19.4 kHz) TABLE XI.



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TABLE XII. VALUES OF FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ISOTROPIC IONOSPHERE ($h_{\rm T}$ * $h_{\rm R}$ = 0, h = 90 km, f * 19.4 kHz)

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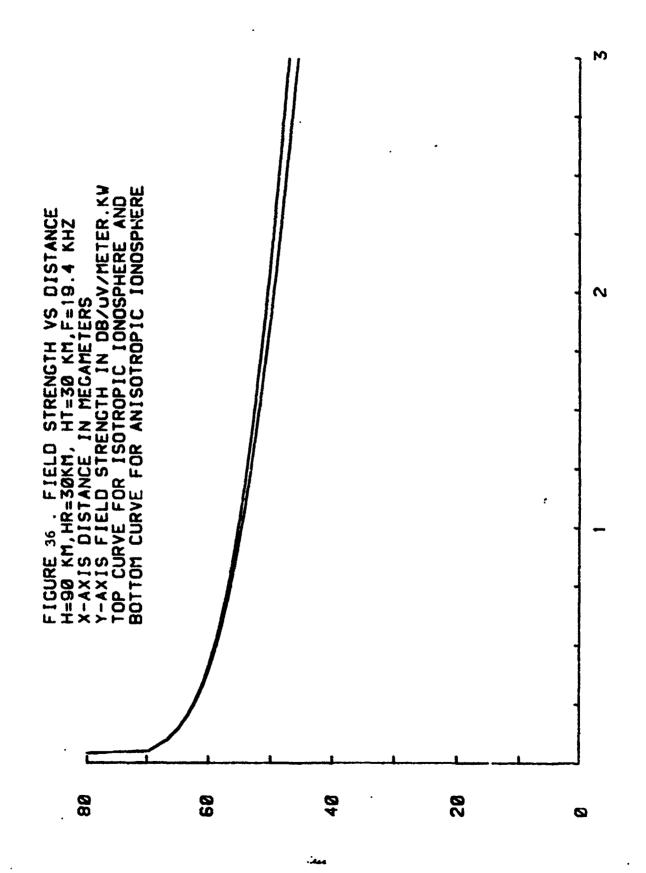
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VALUES OF FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ANISOTROPIC IONOSPHERE ($h_{\rm T}$ = $h_{\rm R}$ = 0, h= 90 km, f = 19.4 kHz) TABLE XIII.

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Ž FIELD STRENGTH DIFFERENCE (DB. UV/METER.KW) VS DISTANCE (NUMBER.X 25 F=19.4 KHZ, HR=0 KM, HT=00 KM, H=90 KM, n=1

TABLE XIV.



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VALUES OF THE FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ISOTROPIC IONOSPHERE ($h_{\rm T}$ = $h_{\rm R}$ = 30 km, h = 90 km, f = 19.4 kHz) XV. TABLE

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VALUES OF THE FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ANISOTROPIC IONOSPHERE ($h_{\rm T}$ = $h_{\rm R}$ = 30 km, h = 90 km, f = 19.4 kHz) TABLE XVI.

8	16 2.8 20 2.9
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VALUES OF THE DIFFERENCE BETWEEN FIRST ORDER MODE FIELD STRENGTH VS DISTANCE FOR ISOTROPIC AND ANISOTROPIC IONOSPHERE. ($h_T=h_R=30~{\rm km},~h=90~{\rm km},~f=19.4~{\rm kHz}$) TABLE XVII.

NADC-83062-40

CONCLUSION

A comprehensive computational analysis of the vertical component of the electric field strength for individual and multimode propagation of electromagnetic waves at very low frequencies (19.4 KHz and 27 KHz) as a function of distance for different ionospheric conditions and transmitting and receiving antenna elevation configurations has been completed. The analysis proves that the field strength value for the individual modes is dependent on the attenuation constant and the modified height gain factor which have been proved to be dependent on the transmitting and receiving antenna elevation configurations. Consequently, at moderately large distances, fields of higher order modes become as significant as that of the first order mode.

The multimode field strength curve displays strong interference phenomenon. The intensity of this interference at a certain distance is dependent upon the transmitting frequency, ionospheric condition and the transmitting and receiving antenna height configurations. When the model interference is strong, the interference pattern displays the occurrence of deep nulls which sometimes occur at distances 800 Km and 2400 Km especially at nighttime when both the antennas are ground-based and the frequency of transmission is 27 KHz.

NADC-83062-40

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PROGRAM FOR THE SINGLE MODE FIELD STRENGTH COMPUTATION APPENDIX A.

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"DO YOU WISH TO CHANGE CONDITIONS AND REPEAT ? (YES,NO)"
"ENTER RECEIVER ALTITUDE IN KM.
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R(N) = 10† ((54.3-10*LCT(A*SIN(D*1.0E-3/A))-K*D+G)/20)
S(N) = 10† ((54.3-10*LGT(A*SIN(D*1.0E-3/A))-L*D+H)/20)
O(N) = 10† ((54.3-10*LGT(A*SIN(D*1.0E-3/A))-L*D+H)/20)
X=0.916-0.407*D
Y=0.809-0.402*D
Z=0.799-0.39*D
                                                                                                                                                                                                                                                                                                                                                     STRENGTH VS DISTANCE"
WINDOW 0,3000,0,80
DIM R(120),S(120),O(120),E(120),E1(120),E2(120)
                                                                                                                                                                                                                                                                         E1 (N) = (R(N) *COS(X) +S(N) *COS(Y) +O(N) *COS(Z)) +Z
E2 (N) = (R(N) *SIN(X) +S(N) *SIN(Y) +O(N) *SIN(Z)) +Z
E(N) = (E) (N) +E2 (N) ) +Ø.5
                                                                                                                                                                                                                                                                                                                                                                   F= 19.4 KHZ"
                                                                                                                                                                                                                                                                                                                                                                                  IN MEGAMETERS"
                                                                                                                                                                                                                                                                                                                                                                                                   STRENGTH
                                                                                                                                                                                                                                                                                                                                                                    HE70 KM, HT=HR=0 KM,
                                                                                                                                                                                                                                                                                                                                                      FIELD
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          50
                                                                                                     <u>-</u>
                                                                                                                                                                                                                                                                                                                                                                                                                                         VIEWPORT 10,122,10,88
AXIS 250,10,0,0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          STEP
                                                                                                     G, H, AND
                                                                                                                                                                                                                                                                                                                                                                                                  FIELD
                                                                                                                                                  TO 120 STEP
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           6000
                                                                                                                                                                                                                                                                                                                                                                                  IX-AXIS
IX-AXIS
                                                                                                     "ENTER
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          10
                                                       K=0.001555
                                                                       L=0.00656
                                                                                      0=0.01519
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          0=50
                                                                                                                                                  FOR N=1
                                           A=6400
                                                                                                                   INPUT
                                                                                                                                                                                                                                                                                                                                                                   PRINT
                                                                                                                                                                                                                                                                                                                                                                                                   PRINT
                                                                                                     PRINT
                                                                                                                                                                                                                                                                                                                                                                                                                 HOME
                                                                                                                                                                                                                                                                                                                                       HOME
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50
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 670
                                                                                                                                                                                                                                                                                                                                                                 IF 0(D/50)=0 THEN (V=20*LGT(0(D/50))
DRAW D, V
D=D+50
IF D=6050 THEN 610
GO TO 550
MOVE 0,130
                                                                                                                                                                                                                                                                                                                                                                                                                                                                            THEN 610
50)=0 THEN T (R(D/50))
                                                                                                                                                       1HEN (150)
                                                                                                                                                      IF S(D/50)=0
T=20*LCT(S(D/
DRAW D, T
D=D+50
    0-(05/0
                                                                                                                                                                                                                                                                                                                0,130
                                                                                                       1,130
                                                                                                                                                                                                                                                              0=6050
                                                                                                                                                                                                                                                                                      TO 460
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  11
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         PRINT
                                                                                                                                                                                                                                                                                                               MOVE
D=50
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  PRIN
                                                                                                      MOVE
D=50
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               PRIN.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     DRAW
                                                                            NEXT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   END
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    FOR
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RETURN

790